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Original Article

Optimization of wind speed and vertical diffusion from source near the ground

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Abstract

In this paper, the advection-diffusion equation was solved in two dimensions to calculate the crosswind integrated concentration by Laplace technique. Taking different two shaped for the wind speed and the eddy diffusivity in stable and unstable conditions (Businger 1973). A comparison between two predicted models, and observed data measured at Prairie Grass (Barad, 1958) diffusion experiment in O'Neill, Nebraska, 1956, has been carried out. One finds that there was a good agreement between our two predicted models and the observed concentrations. Also from the statistical technique, one can conclude that two predicted models perform better with the observed concentrations.

Keywords: Diffusion Equation, Laplace Technique, Wind Speed, Eddy Diffusivity, friction velocity.

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Introduction

Advection-diffusion equation is one of the most important partial differential equations and observed in a wide range of different applications. This equation is solved in three dimensions space (x,y,z) using separation of variables technique to evaluate pollutant concentration per emission rate, taking eddy diffusivities of pollutants and mean wind speed in neutral case by Liu and Hildemann (1996), Singh and Tanaka (2000), Kumar Pramod (2010) and Essa and Sawsan (2015). A crosswind integrated concentration has been evaluated using experimental ground-level concentration provided by Prairie Grass (Barad, 1958) diffusion experiment.

John (2011) has suggested analytical and approximate solutions for the atmospheric dispersion problem under a wide range of simplifying assumptions at boundary conditions. These analytical solutions are especially useful to engineers and environmental scientists who study pollutant transport since they allow parameter sensitivity and source estimation studies to be performed**.** Palazzi *et al.,* (1982) have proposed a simple model for studying the diffusion of substances emitted in steady-state released of short duration assuming the presence of an infinite mixing layer. A combination of diffusion and advection that occurs within the air on the Earth's surface is called dispersion (khaled *et al.*, 2015). A simple scheme to describe vertical diffusion in terms of well-defined surface layer parameters was derived by Van Ulden (1978). In this work, the crosswind integrated concentration of pollutants is estimated by Laplace technique using different shapes of wind speed and eddy diffusivity in stable unstable conditions. A comparison between two predicted models, and observed data measured at Prairie Grass (Barad, 1958) diffusion experiment in O'Neill, Nebraska, 1956, has been carried out. Statistical between two predicted and observed is calculated.

Description Technique

Studying of pollutants dispersion into the atmosphere is depending on the diffusion equation. Advection-diffusion equation of pollutants in the air can be written as (Tiziano and Vilhena 2012):

$$u\,\frac{\partial c(x,y,z)}{\partial x} = \frac{\partial}{\partial y}\left(k_y\frac{\partial c}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z\frac{\partial c}{\partial z}\right) \tag{1}$$

where: c(x,y,z) is the concentration of pollutant (g/m^3), u is the wind speed (m/s), K_y, k_z is the eddy diffusivities in a lateral and vertical direction respectively. By integrating equation (1) with respect to y from $-\infty$ to ∞, then one gets:

$$u\frac{\partial}{\partial x}\int_{-\infty}^{\infty} c(x,y,z)dy = k_y\frac{\partial c(x,y,z)}{\partial y}\bigg|_{-\infty}^{\infty} + k(x)\frac{\partial^2}{\partial z^2}\int_{-\infty}^{\infty} c(x,y,z)dy \tag{2}$$

Let:

$$\int_{-\infty}^{\infty} c(x,y,z)dy = c_y(x,z) \tag{3}$$

$$where \qquad k_y\frac{\partial c(x,y,z)}{\partial y}\bigg|_{-\infty}^{\infty} = 0 \tag{4}$$

By substituting from equation (3) & (4) into equation (2), one gets:

$$u\frac{\partial c_y(x,z)}{\partial x} = \frac{\partial}{\partial z}\left[k(z)\frac{\partial c_y(x,z)}{\partial z}\right] \tag{5}$$

According to the dependence of eddy diffusivity “k” and wind speed profile “u” on the height variable (z), one can solve advection-diffusion equation for non-homogeneous turbulence by the Laplace transform technique, A stepwise approximation have been performed on these coefficients discretizing the height h of the PBL into N sub-intervals in a manner of inside each sub-region, k (z) and u (z), assuming the following average values:

$$k_n = \frac{1}{z_{n+1} - z_n}\int_{z_n}^{z_{n+1}} k_n(z)dz$$

$$u_n = \frac{1}{z_{n+1} - z_n}\int_{z_n}^{z_{n+1}} u(z)dz$$

$$u\frac{\partial c_{y_{n(x,z)}}}{\partial x} = k_n(z)\frac{\partial^2 c_{y_{n(x,z)}}}{\partial z^2}$$

for n=1: N.

Applying the Laplace transform on “x” under the boundary conditions:

$$c_{y_n(0,z_n)} = \frac{Q}{u_n}\delta(z_n - h_s) \tag{i}$$

$$k_n(z)\frac{\partial c_{y_n(x,z)}}{\partial z} = 0 \qquad \text{at} \quad z_n = 0, \mathrm{h} \tag{ii}$$

where "h" is a mixing height.

Then the equation (5) can be written as:

$$\int_0^\infty u\frac{\partial c_{y_n}}{\partial x}e^{-sx}dx = k_n(z)\int_0^\infty \frac{\partial^2 c_{y_n}}{\partial z^2{}_n}e^{-sx}dx \tag{6}$$

Integrating and substituting in the equation (6), one gets:

$$-uc_{y_n}(0,\mathrm{z}) + \mathrm{s}\, c_{y_n}\tilde{c}(\mathrm{s,z}) = k_n(z)\frac{\partial^2 \tilde{c}_{y_n}(\mathrm{s,z})}{\partial z^2{}_n} \tag{7}$$

Applying the boundary condition (i), one gets:

$$\frac{\partial^2 \tilde{c}_{y_n}(\mathrm{s,z})}{\partial z^2{}_n} - \frac{su}{k_n}\tilde{c}_{y_n}(\mathrm{s,z}) = -\frac{Q}{k_n}\delta(z_n - h_s) \tag{8}$$

Now applying Laplace transform on z then:

$$p^2\tilde{\tilde{c}}_{y_n}(s,p) - pc_{y_n}(s,0) - \frac{\partial \tilde{c}_{y_n}(s,0)}{\partial z} - \frac{us}{k_n}\tilde{\tilde{c}}_{y_n}(s,p) = -\frac{Q}{k_n}e^{-ph_s} \tag{9}$$

Substituting the condition (ii), equation (9) becomes:

$$\tilde{\tilde{c}}_{y_n}(s,p) = \frac{c_{y_n}(s,0)p}{(p^2-\frac{us}{k_n})} - \frac{Qe^{-ph_s}}{k_n(p^2-\frac{us}{k_n})} \tag{10}$$

$$\tilde{\tilde{c}}_{y_n}(s,p) = c_{y_n}(s,0)F(s,p) - \frac{Q}{k_n}e^{-ph_s}G(s,p) \tag{11}$$

where $\quad \mathrm{F}\,(\mathrm{s,p}) = \frac{p}{(p^2-\frac{us}{k_n})} \quad$ and $\quad G(s,p) = \frac{1}{(p^2-\frac{us}{k})}$

The inverse of Laplace transform on "z" is taken i.e. $L^{-1}\{(\tilde{\tilde{c}}_y(s,p), z\} = \tilde{c}_{y_n}(s,z)$

$$\tilde{c}_{y_n}(s,z) = \frac{c_{y_n}(s,0)}{2}\left[e^{\sqrt{\frac{su}{k_n}}z} + e^{-\sqrt{\frac{su}{k_n}}z}\right] - \frac{Q}{2k_n}\sqrt{\frac{k_n}{su}}\left[e^{\sqrt{\frac{su}{k_n}}(z-h_s)} - e^{-\sqrt{\frac{su}{k_n}}(z-h_s)}\right]H(z-h_s) \tag{12}$$

Let $\quad R_n = \sqrt{\frac{su}{k_n}} \quad$ and $\quad R_a = \sqrt{suk_n}$

$$\tilde{c}_{y_n}(s,z) = \frac{c_{y_n}(s,0)}{2}[e^{R_n z} + e^{-R_n z}] - \frac{Q}{2R_a}\left[e^{R_n(z-h_s)} - e^{-R_n(z-h_s)}\right]H(z-h_s) \tag{13}$$

$$\tilde{c}_{y_n}(s,z) = c_{y_n}(s,0)\cosh R_n z - \frac{Q}{R_a}\sinh R_n(z-h_s) * H(z-h_s) \tag{14}$$

Applying the boundary condition (ii) one gets:

$k_n(z)\frac{\partial}{\partial z}\tilde{c}_{y_n}(s,z) = 0$ at $z = \mathrm{h}$ then:

$$\frac{\partial}{\partial z}\tilde{c}_{y_n}(s,z) = R_n c_{y_n}(s,0)\sinh R_n z - \frac{Q}{R_a} R_n \cosh R_n(z-h_s)H(z-h_s) - \frac{Q}{R_a}\sinh R_n(z-h_s)\frac{\partial}{\partial z}H(z-h_s) \quad (15)$$

$$c_{y_n}(s,0)\sinh(R_n \mathrm{h}) = \frac{Q}{R_a}\cosh\big(R_n(\mathrm{h}-h_s)\big)H(\mathrm{h}-h_s) \quad (16)$$

$$c_{y_n}(s,0) = \frac{Q}{R_a}\frac{\cosh R_n(\mathrm{h}-h_s)}{\sinh(R_n \mathrm{h})}$$

$$c_{y_n}(s,0) = \frac{Q}{\sqrt{suk_n}}\frac{\cosh\sqrt{\frac{su}{k_n}}(\mathrm{h}-h_s)}{\sinh\sqrt{\frac{su}{k_n}}\mathrm{h}} \quad (17)$$

Substituting from equation (17) in equation (14) then one gets:

$$\tilde{c}_{y_n}(s,z) = \frac{Q}{\sqrt{suk_n}}\frac{\cosh\sqrt{\frac{su}{k_n}}(\mathrm{h}-h_s)}{\sinh\sqrt{\frac{su}{k_n}}\mathrm{h}}\cosh R_n z - \frac{Q}{R_a}\sinh R_n(z-h_s) * H(z-h_s)$$

At the ground level (i.e. z=0), the following equation can be describing crosswind integrated concentration as (Essa *et al.*, 2017):

$$\tilde{c}_{y_n}(s,0) = \frac{Q}{\sqrt{suk_n}}\frac{\cosh\sqrt{\frac{su}{k_n}}(\mathrm{h}-h_s)}{\sinh\sqrt{\frac{su}{k_n}}\mathrm{h}} \quad \text{at } z=0 \quad (18)$$

By using Gaussian quadrature formulas one gets:

$$\frac{c_{y_n}(x,z)}{Q} = \sum_{i=1}^{8} a_i\left(\frac{p_i}{x}\right)\frac{1}{\sqrt{\frac{uk_n(z)p_i}{x}}}\frac{\cosh\sqrt{\frac{p_i u}{xk_n}}(z_i-h_s)}{\sinh\sqrt{\frac{p_i u}{xk_n}}z_i} \quad (19)$$

<u>For stable case</u>: $1/L \geq 0$, where L is a Monin-Obukhov length scale. The wind speed is taken from Van Ulden (1978) as follows:

$$u = \left(\frac{u_*}{k}\right)\left[Ln\left(\frac{z}{z_o}\right) - \psi\left(\frac{z}{L}\right)\right]$$

where k =0.35 [Von-Karman constant], u_* is the friction velocity, z_o=0.008 m [the surface roughness length]

and ψ is given as follows:

$$\psi = -4.7z/L$$

Then,

$$u = \left(\frac{u_*}{k}\right)\left[Ln\left(\frac{z}{z_o}\right) + \frac{4.7z}{L}\right]$$

Also, the eddy diffusivity is given from Van Ulden (1978) as follows:

$$K_n = ku_*z/\phi_n(\frac{z}{L})$$

where ϕ_n is given from Van Ulden (1978) as follows:

$$\phi_n = 0.74(1 + \frac{6.3z}{L})$$

So,

<u>For unstable case</u>: $1/L \leq 0$, then the wind speed in the form (Businger 1973):

$$u = \left(\frac{u_*}{k}\right)[Ln\left(\frac{z}{z_o}\right) - \psi(\frac{z}{L})$$

where ψ is the form:

$$\psi = 2Ln[\frac{(1+\chi)}{2}] + Ln[\left(\frac{(1+\chi^2)}{2}\right] - 2tan^{-1}(\chi) + \pi/2$$

$$\chi = (1 - \frac{15z}{L})^{1/4}$$

Also the eddy diffusivity in the form (Businger 1973):

$$K_n = ku_*z/\phi_n(\frac{z}{L})$$

where ϕ_n in the form:

$$\phi_n = 0.74(1 - \frac{9\,z}{L})^{-1/2}$$

Results and Discussion

The performance of the observed and predicted crosswind integrated concentration has been evaluated using experimental ground-level concentration provided by Prairie Grass (Barad, 1958) diffusion experiment. The prairie Grass experiment was realized in O'Neill, Nebraska, 1956. The pollutant Sulphur dioxide (SO_2) at a height of 1.5m in three downwind distances (50, 200 and 800m). The Prairie Grass site was flat with a roughness length 0.008m. The meteorological parameters and concentration measured during the prairie Grass stable experiment. L, Q, h, u_* and c_y are Monin-Obukhov length, the emission rate, mixing height, friction velocity and the ground–level crosswind integrated concentration respectively in stable and unstable conditions respectively are shown in Tables 1 and 2 respectively.

The two figures of the predicted and observed crosswind integrated concentration for three downwind distances 50, 200 and 800m respectively in Prairie Grass in stable and unstable stabilities are shown in figures one and two. From the two figures, one finds that the predicted crosswind integrated concentrations are inside one to one in stable and unstable conditions except for some points at downwind distance at 800m in stable condition and downwind distance at 50m in an unstable condition which lie inside a factor of two. Also, this result is obtained by the statistical method.

Model evaluation statistics

The statistical technique between predicted and observed crosswind integrated concentration for three downwind distances 50, 200 and 800m respectively were performed (Hanna 1989). The following standard statistical performance measures that characterize the agreement between prediction ($C_p = C_{pred}$) and observations ($C_o = C_{obs}$):

$$\text{Fraction Bias (FB)} = \frac{(\overline{C_o} - \overline{C_p})}{[0.5(\overline{C_o} + \overline{C_p})]}$$

$$\text{Normalized Mean Square Error (NMSE)} = \frac{\overline{(C_p - C_o)^2}}{\overline{(C_p C_o)}}$$

$$\text{Correlation Coefficient (COR)} = \frac{1}{N_m}\sum_{i=1}^{N_m}(C_{pi} - \overline{C_p}) \times \frac{(C_{oi} - \overline{C_o})}{(\sigma_p \sigma_o}$$

$$\text{Factor of Two (FAC2)} = 0.5 \leq \frac{C_p}{C_o} \leq 2.0$$

where σ_p and σ_o are the standard deviations of predicted $\overline{C_p}$ and observed $\overline{C_o}$ normalized crosswind integrated concentration respectively. Over bars indicate the average over all measurements. For a perfect model NMSE must be = FB = 0 and COR= FAC_2 = 1.

Table (1) Meteorological parameters and concentration measured during the prairie Grass experiment in stable condition

Run	L(m)	Q(g/s)	h(m)	U_* (m/s)	C_y (gm^{-2}) observed			C_y (gm^{-2}) predicted		
					x=50m	x=200 m	X=800	X=50 m	X=200m	X=800m
13	3.4	61.1	23	0.09	38	223	133	25	181	153
14	1.6	49.1	12	0.05	153	153	31	146	141	26
17	48	56.5	131	0.21	105	34	11	95	39	9
18	25	57.6	92	0.2	108	46	20	114	45	18
21	172	50.9	333	0.38	58	18	6	57	19	6
22	204	48.4	400	0.46	47	14	4	48	15	3
23	193	40.9	358	0.39	47	17	4	45	20	3.8
24	248	41.2	400	0.38	47	15	4	46	16	3.5
28	24	41.7	81	0.16	136	51	15	136	50	13
29	36	41.5	119	0.23	99	38	12	100	39	13
32	8.3	41.4	43	0.13	159	115	56	153	111	53
35	53	38.8	147	0.24	88	32	10	87	31	9
36	7.8	40.0	36	0.1	193	100	41	192	99	41
37	95	40.3	216	0.29	61	21	7	60	20	6
38	99	45.4	217	0.28	78	26	8	76	25	7
39	9.8	40.7	48	0.14	112	39	---	112	38	15
40	8	40.5	39	0.11	115	43	17	116	42	18
41	35	39.9	117	0.23	79	32	12	77	32	11
42	129	56.4	275	0.37	52	17	5	53	16	5
46	114	99.7	257	0.34	63	23	7	61	21	3
53	10	45.2	54	0.17	154	83	32	150	81	31
54	40	43.4	128	0.24	81	30	11	82	29	10.5
55	124	45.3	279	0.37	53	18	5	51	16	6
56	76	45.9	194	0.29	71	24	7	69	25	6
58	6.4	40.5	35	0.11	161	105	51	160	102	49
59	11	40.2	51	0.14	140	81	31	139	81	30
60	58	38.5	166	0.28	62	23	8	61	22	7

Table (2) Meteorological parameters and concentration measured during the prairie Grass unstable experiment

Run	-L(m)	Q(g/s)	h(m)	u_* (m/s)	C_y (gm^{-2}) observed			C_y (gm^{-2}) predicted		
					X=50m	x=200m	X=800	X=50m	X=200m	X=800m
1	9	82	260	0.19	7	1.51	0.062	2.5	0.13	0.022
5	28	78	780	0.39	3.30	0.81	0.092	3.2	0.54	0.014
7	10	90	1340	0.31	4.00	1.00	0.18	4.5	1.7	0.21
8	18	91	1380	0.31	5.10	1.10	0.14	4.6	0.91	0.19
9	31	92	550	0.46	3.70	1.00	0.13	3.2	0.86	0.14
10	11	92	950	0.32	4.50	0.71	0.032	4.6	0.86	0.033
15	8	96	80	0.23	7.10	1.35	0.11	6.7	1.26	0.14
16	5	93	1060	0.24	5.00	0.48	0.017	6.2	0.52	0.015
19	28	102	650	0.39	4.50	0.86	0.058	4.1	0.84	0.060
20	62	102	710	0.60	3.40	0.85	0.13	2.7	0.56	0.12
25	6	104	650	0.20	7.90	0.75	0.063	8.4	0.72	0.065
26	32	98	900	0.43	3.90	1.04	0.127	3.6	0.98	0.128
27	30	99	1280	0.42	4.30	1.16	0.176	3.4	1.15	0.167
30	39	98	1560	0.46	4.20	1.11	0.10	4.5	1.15	0.12
43	16	99	600	0.35	5.0	1.09	0.12	4.5	1.03	0.14
44	25	101	1450	0.40	4.50	1.09	0.14	4.0	1.07	0.18
49	28	102	550	0.45	4.30	1.16	0.15	4.0	1.17	0.19
50	26	103	750	0.44	4.20	0.91	0.11	3.7	0.75	0.16
51	40	102	1880	0.45	4.70	1.00	0.084	4.5	1.05	0.092
61	38	102	450	0.51	3.50	1.14	0.20	3.2	1.16	0.22

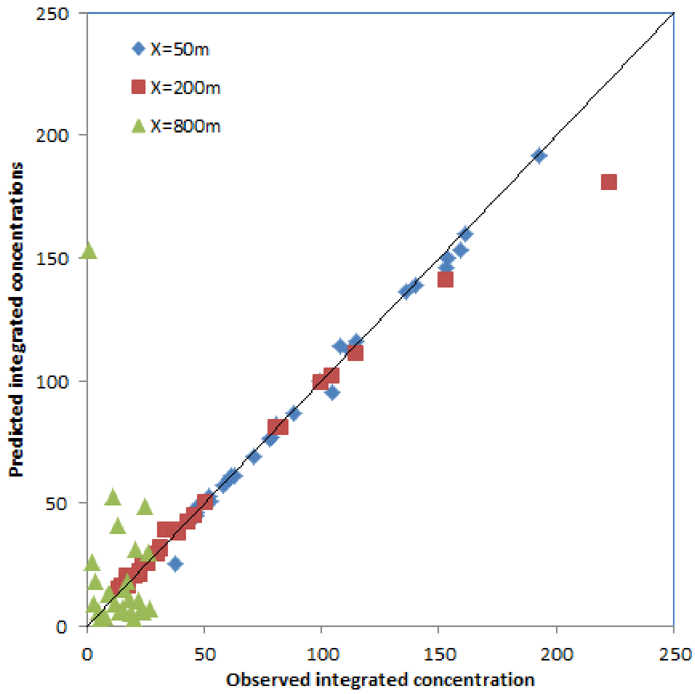

Fig (1) Comparison between the predicted and observed integrated concentration for three downwind distances 50, 200 and 800m respectively in Prairie Grass in <u>stable</u> condition

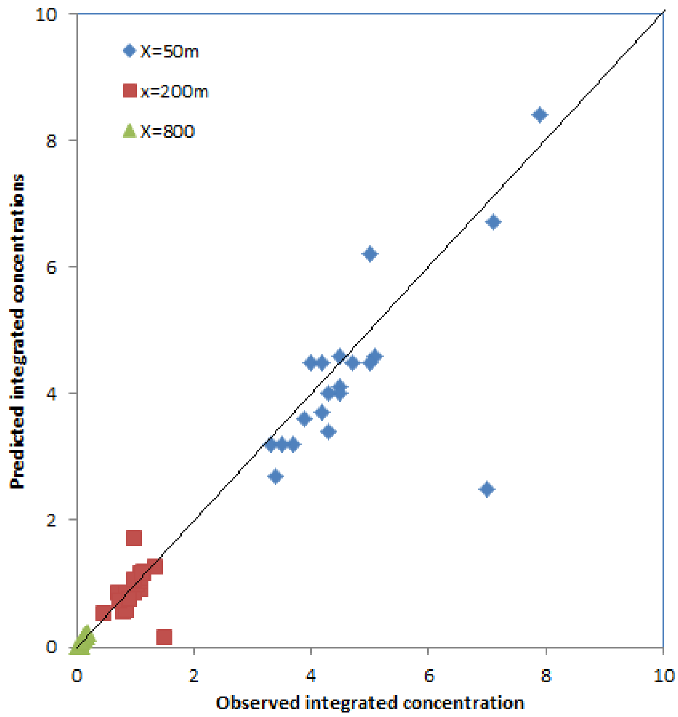

Fig (2) Comparison between the predicted and observed integrated concentration for three downwind distances 50, 200 and 800m respectively in Prairie Grass in <u>unstable</u> condition

Table 3: Statistical between predicted and observed integrated crosswind concentrations at 50, 200 and 800m in stable condition

Models	NMSE	FB	COR	FAC_2
Predicted 1(50m)	0.00	0.02	1	0.97
Predicted 2 (200m)	0.03	0.05	0.99	0.99
Predicted 2 (800m)	0.04	0.01	1	0.93

Table 4: statistical between predicted and observed integrated crosswind concentrations at 50, 200 and 800m in unstable condition

Models	NMSE	FB	COR	FAC_2
Predicted 1(50m)	0.06	0.09	0.68	0.92
Predicted 2 (200m)	0.14	0.09	0.21	0.94
Predicted 2 (800m)	0.07	-0.08	0.90	1.04

From the statistical method, one finds that all models are inside a factor of two with observed data. The correlations of the two predicted at 50 and 200m in stable condition are very better than the cross ponding in in unstable condition. Regarding NMSE and FB, three predicted models were performance well with observed data in stable and unstable conditions.

Conclusions

In this work, the advection-diffusion equation is solved in two dimensions to calculate the crosswind integrated concentration by Laplace technique. Different two shaped for the wind speed and the eddy diffusivity in stable and unstable conditions are taken (Businger 1973). A comparison between two predicted models, and observed data measured at Prairie Grass (Barad, 1958) diffusion experiment in O'Neill, Nebraska, 1956, has been carried out. One finds that the predicted crosswind integrated concentrations are inside one to one in stable and unstable conditions except for some points at downwind distance at 800m and 50m in stable and unstable conditions which lie inside a factor of two. Also from the statistical technique, one can conclude that three predicted models perform better with the observed concentrations.

References

Barad M. L. (Ed) (1958). Project Prairie Grass, A field program in diffusion, vol. 1., Geophysics Research Paper no. 59, Geophysics Research Directorate, Air Force Cambridge Research Center.

Busnger J.A. (1973) In workshop of Micrometeorology, Haugen D. A. (ED), American Meteorological Society.

Hanna, S. R., 1989, “confidence limit for air quality models as estimated by bootstrap and Jacknife resembling methods”, Atom. Environ. 23, 1385-1395.

John M. Stock I. E. (2011). The Mathematics of Atmospheric Dispersion Modeling. SIAM Review, Vol. 53. No. 2, pp. 349-372.

Khaled S. M. Essa and Sawsan. E. M. Elsaid. (2015): “Solving the Advection-Diffusion Equation in Three Dimensions in Neutral Case”, Pyrex Journal of Ecology and the Natural Environment Vol 1 (1) pp. 001-006.

Khaled S. M. Essa, Sawsan E. M. Elsaid and Fawzia Mubarak (2015), "Time-Dependent Advection-Diffusion Equation in Two Dimensions" Journal of Atmosphere, 1(1): 8-16.

Khaled S.M. Essa, Aziz N. Mina, Hany S. Hamdy, Fawzia Mubarak and Ayman A. khalifa (2017) under publication in NRIAG Astronomy and geophysics

Lin, J .S., Hildemann, L.M., (1996): "Analytical Solutions of the Atmospheric Diffusion Equation with Multiple Sources and Height-Dependent Wind Speed and Eddy Diffusivities", Atmospheric Environment 30, 239-254.

Palazzi, E., De Faveri, M., Fumarola, G., Ferraiolla, G., (1982), *Diffusion from a steady source of short duration*, Atmospheric Environment 16, 2785–2790.

Pramod Kumar (2010): “Analytical Models for Dispersion of Pollutants in the Atmospheric Boundary Layer”, Ph.D., Indian Institute of Technology Delhi, Indian.

Singh K. M, Tanaka M. (2000): “On Exponential Variable Transformation Based Boundary Element Formulation for Advection-Diffusion Problems”, Eng Anal Bound Elem; 24: 225-235.

Tirabassi Tiziano and Vilhena Marco T. (2012): "Advection-Diffusion in the Atmosphere: Equations and Solutions", Atmospheric Flow Field: Theory, Numerical Methods and Software Tools, 153-173.

Van Ulden A. P. (1978): " Simple estimates for vertical diffusion from sources near the ground", Atmospheric Environment Vol. 12, pp 2125-2129.